Monogamy of nonlocal quantum correlations

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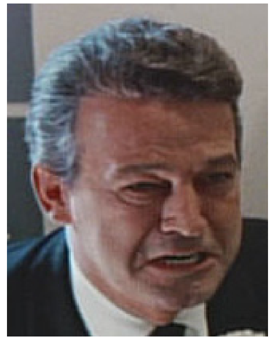
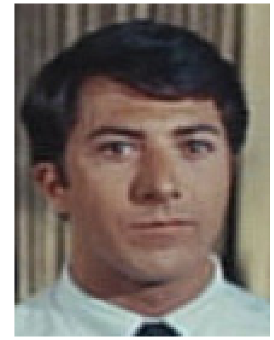
Monogamy of entanglement

Alice



Bob



Charlie

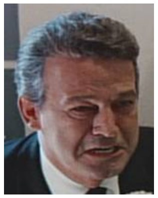
$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$\rho_{ABC} = |\psi\rangle_{AB}\langle\psi|_{AB} \otimes \rho_C$$

1. Can make this quantitative [CoffmanKunduWootters00], [OsborneVerstraete04];
2. Related to security of quantum key distribution, e.g. Ekert scheme [Ekert91].

Classical correlations are not monogamous

Alice



Bob

Charlie

1. The parties share randomness λ.
2. Each party has 0,1 random variables:

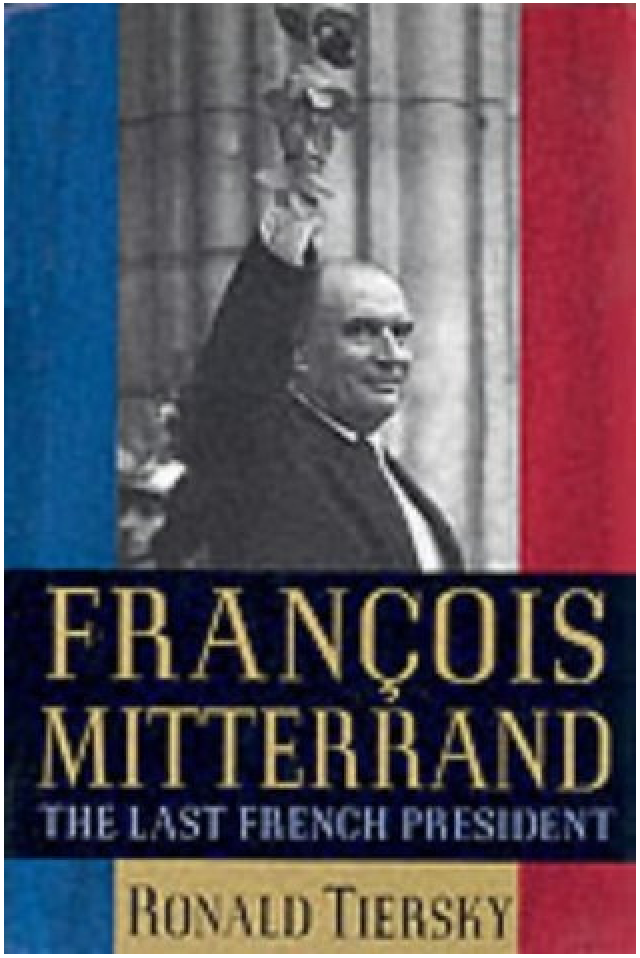
   $\{A_i\}$ $\{B_j\}$ $\{C_k\}$

   that depend on λ and also private randomness.

Joint distribution of AB: $\Pr(A_i \wedge B_j) = \sum_\lambda A_i(\lambda) B_j(\lambda)$ does not restrict joint distribution of AC (except for the trivial requirement the marginals $\Pr(A_i)$ are consistent).

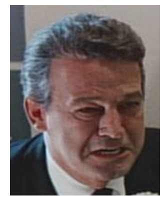
Classical correlations are not monogamous

Neither are the French



Monogamy of quantum correlations
nonlocal

Alice



Bob

Charlie

1. The parties share a quantum state ρ.
2. Each party has ± 1-valued observables:

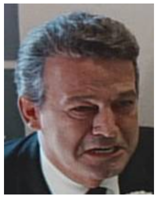
$$\{\mathbf{A}_i\} \qquad \{\mathbf{B}_j\} \qquad \{\mathbf{C}_k\}$$

Joint distribution of AB: $\langle \mathbf{A}_i \mathbf{B}_j \rangle = \mathrm{tr}\,(\rho_{\mathsf{ABC}} \mathbf{A}_i \otimes \mathbf{B}_j)$
can restrict which joint distributions of AC are allowed.

A prerequisite:
Correlations cannot have a classical description.

Example 1: CHSH correlations

Alice



Bob

Charlie

1. The parties share a quantum state ρ (of arbitrary dimension).
2. Each party has measures one of two ± 1-valued observables:

$$\{\mathbf{A}_0, \mathbf{A}_1\} \qquad \{\mathbf{B}_0, \mathbf{B}_1\} \qquad \{\mathbf{C}_0, \mathbf{C}_1\}$$

$$\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle = \text{tr}\left(\rho\left[\mathbf{A}_0\left(\mathbf{B}_0 + \mathbf{B}_1\right) + \mathbf{A}_1\left(\mathbf{B}_0 - \mathbf{B}_1\right)\right]\right)$$

$$\left\langle \mathcal{B}^{\text{AC}}_{\text{CHSH}} \right\rangle = \text{tr}\left(\rho\left[\mathbf{A}_0\left(\mathbf{C}_0 + \mathbf{C}_1\right) + \mathbf{A}_1\left(\mathbf{C}_0 - \mathbf{C}_1\right)\right]\right)$$

Bell Inequality violation

$$\langle \mathcal{B}_{\text{CHSH}} \rangle_c \leq 2$$

$$\max \langle \mathcal{B}_{\text{CHSH}} \rangle_q = 2\sqrt{2}$$

Is there a trade-off between $\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle$ and $\left\langle \mathcal{B}^{\text{AC}}_{\text{CHSH}} \right\rangle$?

Theorem: $\left|\left\langle \mathcal{B}^{\mathsf{AB}}_{\mathsf{CHSH}} \right\rangle\right| + \left|\left\langle \mathcal{B}^{\mathsf{AC}}_{\mathsf{CHSH}} \right\rangle\right| \leq 4$.

[Suggested by Michael Nielsen; see also [ScaraniGisin01].]

Corollary: Suppose $N+1$ parties A, B_1, B_1,. . . , B_N share a quantum state and each chooses to measure one of two observables. Then A violates the CHSH inequality with at most one of the B_i.

See also [MasanesAcınGisin05].

Technique

- Generally, hard to obtain bounds on the quantum value of a nonlocal game.

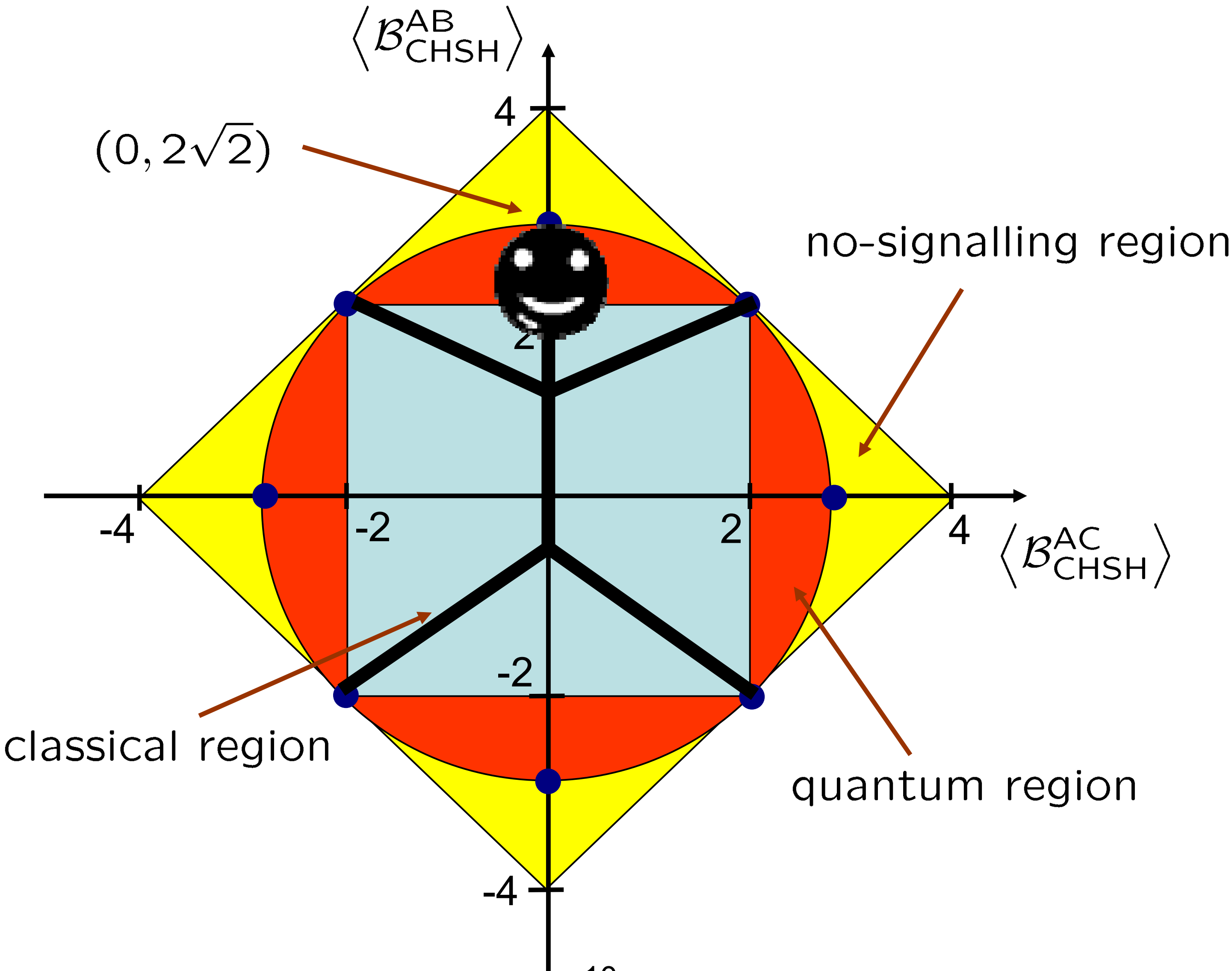
- Quantum correlations are no-signalling.

- So relax to no-signalling probability distributions.

- Determining the no-signalling value of a nonlocal game can be formulated as a linear program.

- For appropriate 3 party version of CHSH game, construct solution to dual program to get bound.

Monogamy of CHSH correlations



Theorem: $\left|\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle\right| + \left|\left\langle \mathcal{B}^{\text{AC}}_{\text{CHSH}} \right\rangle\right| \leq 4.$

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Corollary: Suppose $N+1$ parties A, B_1, B_1,. . . , B_N share a quantum state and each chooses to measure one of two observables. Then A violates the CHSH inequality with at most one of the B_i.

See also [MasanesAcınGisin05].

Theorem: $\left|\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle\right|^2 + \left|\left\langle \mathcal{B}^{\text{AC}}_{\text{CHSH}} \right\rangle\right|^2 \leq 8.$

[Joint work with Frank Verstraete.]

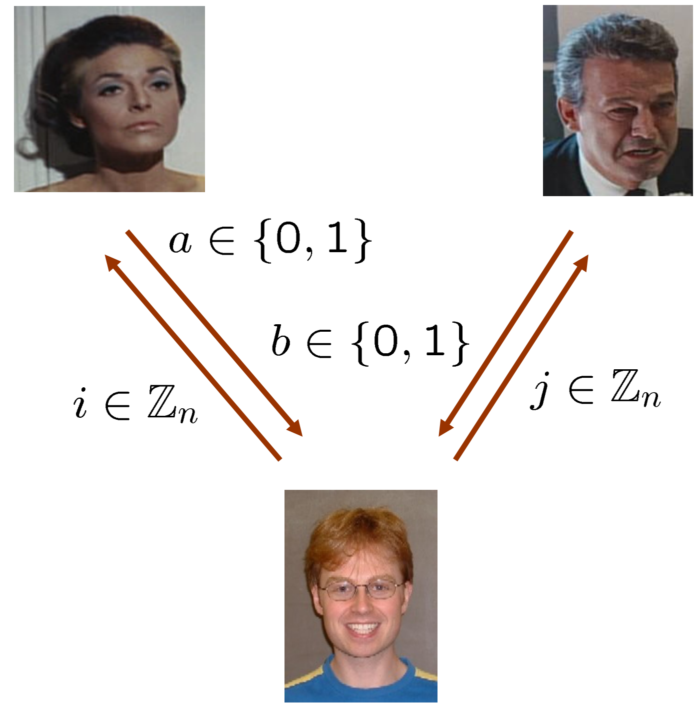
Corollary [Tsirelson]: $\left|\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle\right|^2 \leq \left|\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle\right|^2 + \left|\left\langle \mathcal{B}^{\text{AC}}_{\text{CHSH}} \right\rangle\right|^2 \leq 8$

Therefore, $\left|\left\langle \mathcal{B}^{\text{AB}}_{\text{CHSH}} \right\rangle\right| \leq 2\sqrt{2}.$

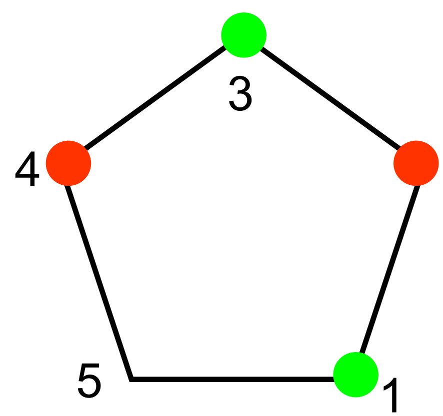
Odd cycle game

The odd cycle game G_{OC} is defined as follows:

1. We choose an integer $i \in \mathbb{Z}_n$ and send it to Alice. With probability $1/2$ we send $j = i$ to Bob, with probability $1/2$ we send $j = i + 1 \mod n$.
2. Alice responds with a bit a and Bob a bit b.
3. They win if $a \oplus b = [i \neq j]$.

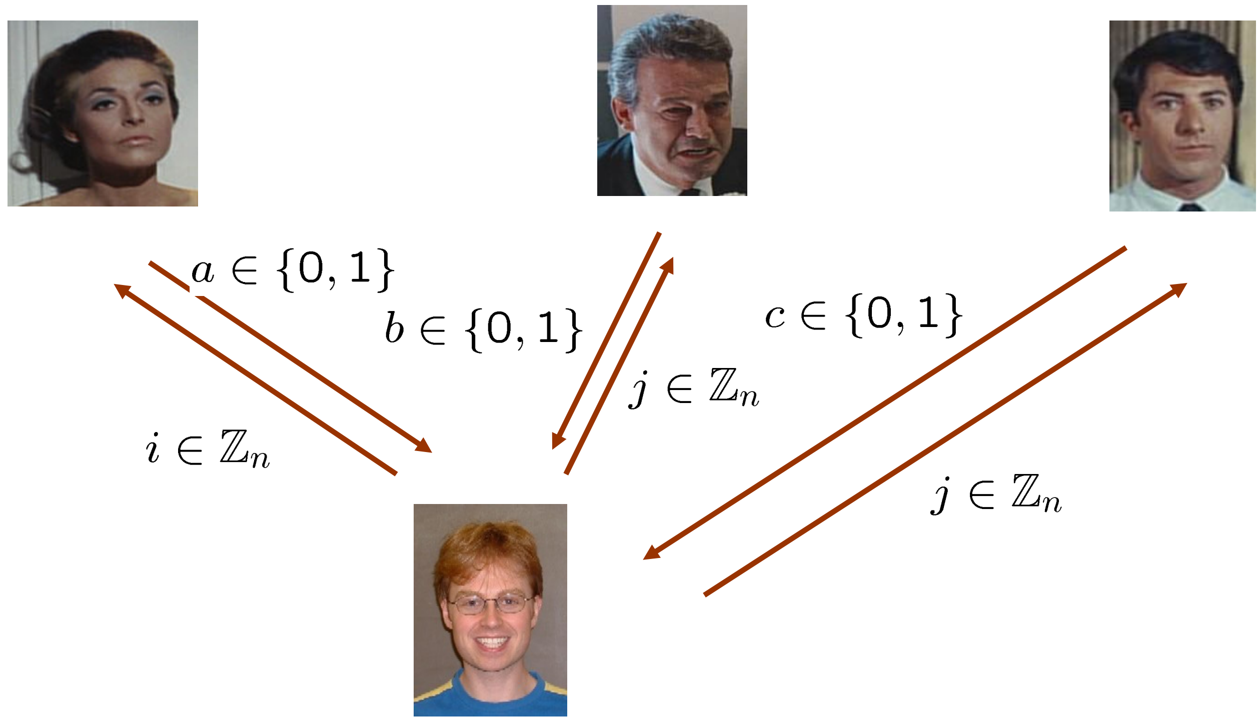


- Classical value $\omega_c(G_{OC}) = 1 - \frac{1}{2n}$.
- Quantum value $\omega_q(G_{OC}) = \cos^2(\pi/4n) \approx 1 - O(\frac{1}{n^2})$.



[CleveHøyerT.Watrous04]

Modified odd cycle game



- Send B's question to additional prover Charlie.
- Players win if (i) A and B win original game, and (ii) B and C agree.

- Classical value $\omega_c(G'_{OC}) = 1 - \frac{1}{2n}$.
- Quantum value $\omega_q(G'_{OC}) = 1 - \frac{1}{2n}$.

These correlations are same as those used in cryptographic scheme [BarrettHardyKent04].

Conclusions

- Described new technique to find Tsirelson bounds on the quantum value of a nonlocal game.
- Demonstrated how to use this technique to quantify the monogamy of quantum correlations.

Further work

- What is the power of MIP_{ns}?
- How many extra provers are required to prevent entangled provers from cheating?

Thanks to Richard Cleve, Michael Nielsen, John Preskill, Graeme Smith, Wim van Dam, Frank Verstraete, John Watrous.