Simulating quantum correlations as a sampling problem

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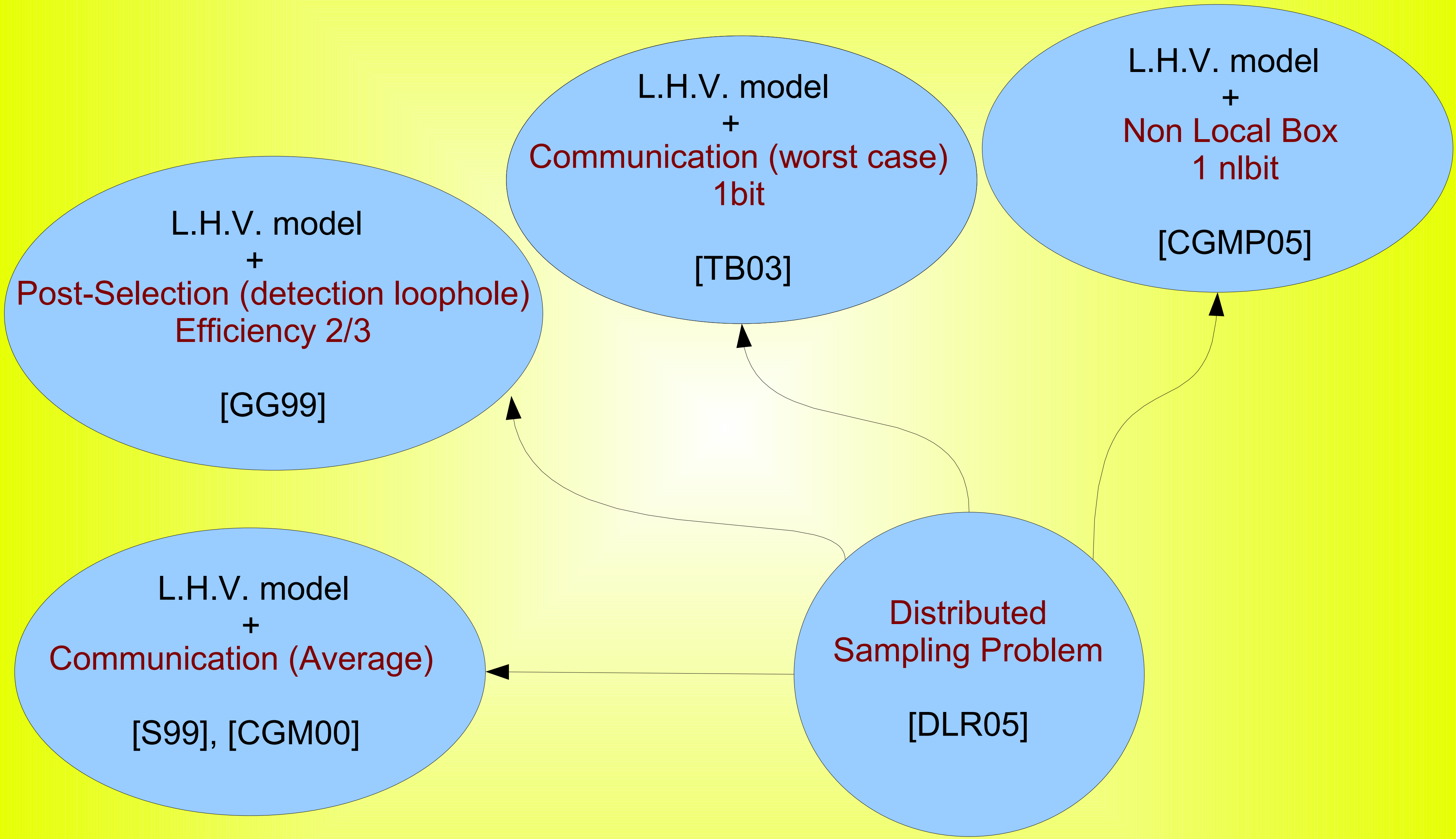
The problem of simulating quantum correlations



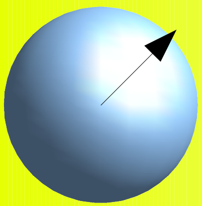
Local Hidden Variable Model

λ

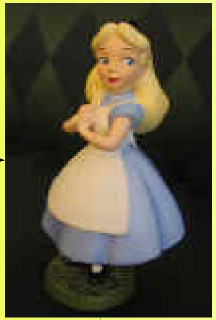
Doesn't depend on the inputs!

Input:

$\vec{a}$

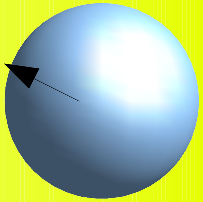


Alice



Input:

$\vec{b}$



Bob

Output: $A(\vec{a},\lambda) \in \{-1,+1\}$

Output: $B(\vec{b},\lambda) \in \{-1,+1\}$

Bell's theorem: ***impossible to reproduce quantum correlations***

Biased Hidden Variable Model

Infinite biased Shared Randomness

$$\vec{\lambda}_s \in S_2 \quad \vec{\lambda}_s \sim \frac{\left|\vec{a}\cdot\vec{\lambda}_s\right|}{2\pi}$$

$\vec{a}$ $\vec{\lambda}_s$ $\vec{\lambda}_s$ $\vec{b}$

Alice

Bob

$$A(\vec{a},\vec{\lambda}_s) = -sign(\vec{a}\cdot\vec{\lambda}_s)$$

$$B(\vec{b},\vec{\lambda}_s) = sign(\vec{b}\cdot\vec{\lambda}_s)$$

$$P(A,B) = \frac{(1 - AB \ \ \vec{a}\cdot\vec{b})}{4}$$

Step 1: Local Sampling of the biased distribution: $\vec{\lambda}_s \sim \frac{|\vec{a}\cdot\vec{\lambda}_s|}{2\pi}$

Shared randomness independent on the inputs : $\vec{\lambda}_0, \vec{\lambda}_1, \vec{\lambda}_2, \ldots, \vec{\lambda}_k, \ldots \sim U_{S_2}$

The rejection method:

Set k=0

1. Alice picks $\vec{\lambda}_k \sim U_{S_2}$

2. Alice picks $u_k \sim U_{[0,1]}$

3. Test whether $u_k \leq |\vec{a}\cdot\vec{\lambda}_k|$

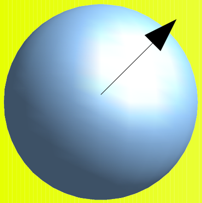
If test succeeds, Alice **ACCEPTS** $\vec{\lambda}_k$ and sets $\vec{\lambda}_s = \vec{\lambda}_k$

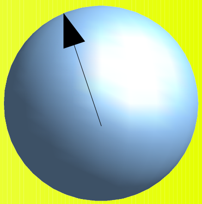
Otherwise, Alice **REJECTS** $\vec{\lambda}_k$ Go back to 1 with k=k+1

When the process terminates, Alice has $\vec{\lambda}_s \sim \frac{|\vec{a}\cdot\vec{\lambda}_s|}{2\pi}$

Step 2: Distributed Sampling problem With communication

$$\vec{\lambda}_0, \vec{\lambda}_1, \ldots \vec{\lambda}_k \sim U_{S_2}$$


$\vec{a}$


$\vec{b}$

Alice

Bob

Index k

Communication

2 bits on average

[Steiner99 + Gisin² 00]

$\vec{\lambda}_s = \vec{\lambda}_k$

Rejection method :

Set k= 0

1. **Alice picks** $\vec{\lambda}_k \sim U_{S_2}$
2. **Alice picks** $u_k \sim U_{[0,1]}$
3. **Test If** $u_k \le |\vec{a} \cdot \vec{\lambda}_k|$

→ $\vec{\lambda}_s = \vec{\lambda}_k$

→ **Go to 1. k=k+1**

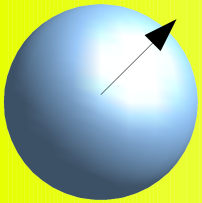
$\vec{\lambda}_s$

$$\vec{\lambda}_s \sim \frac{|\vec{a} \cdot \vec{\lambda}_s|}{2\pi}$$

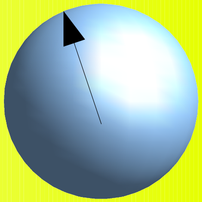
$\vec{\lambda}_s$

Step 2: Distributed Sampling problem With post selection

$$\vec{\lambda}_0, \vec{\lambda}_1, \ldots \vec{\lambda}_k \sim U_{S_2}$$



$\vec{a}$



$\vec{b}$

Alice

Bob

Post selection

[Gisin Gisin 00]

$\vec{\lambda}_s = \vec{\lambda}_0$

Rejection method :

1. Alice picks $\vec{\lambda}_0 \sim U_{S_2}$

2. Alice picks $u_0 \sim U_{[0,1]}$

3. Test If $u_0 \leq |\vec{a} \cdot \vec{\lambda}_0|$

→ $\vec{\lambda}_s = \vec{\lambda}_0$

→  **Abort**

P(output)= 1/2

$\vec{\lambda}_s$

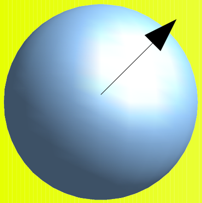
$$\vec{\lambda}_s \sim \frac{|\vec{a} \cdot \vec{\lambda}_s|}{2\pi}$$

$\vec{\lambda}_s$

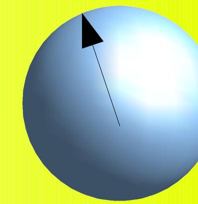
But we can be more clever...

Used only by Alice !

$$\vec{\lambda}_0, \vec{\lambda}_1, \ldots \vec{\lambda}_k \sim U_{S_2}$$



$\vec{a}$



$\vec{b}$

Alice

Bob

Post selection

[Gisin Gisin 00]

$\vec{\lambda}_s = \vec{\lambda}_0$

Rejection method :

1. Alice picks $\vec{\lambda}_0 \sim U_{S_2}$

2. Alice picks $u_0 \sim U_{[0,1]}$

3. Test If $u_0 \leq |\vec{a} \cdot \vec{\lambda}_0|$

→ $\vec{\lambda}_s = \vec{\lambda}_0$

→  **Abort**

P(output)= 1/2

$\vec{\lambda}_s$

$$\vec{\lambda}_s \sim \frac{|\vec{a} \cdot \vec{\lambda}_s|}{2\pi}$$

$\vec{\lambda}_s$

Local Sampling of the biased distribution: a new method

Recall the rejection method:

1. Alice picks $\vec{\lambda}_0 \sim U_{S_2}$

2. Alice picks $u_0 \sim U_{[0,1]}$

3. Test whether $u_0 \leq \left|\vec{a} \cdot \vec{\lambda}_0\right|$

If **Test OK** Alice **ACCEPTS** $\vec{\lambda}_0$ and sets $\vec{\lambda}_s = \vec{\lambda}_0$

Otherwise Alice **REJECTS** $\vec{\lambda}_0$ go back to 1 with another

So, Alice has $\vec{\lambda}_s \sim \left|\vec{a} \cdot \vec{\lambda}_s\right| / 2\pi$

Local Sampling of the biased distribution: a new method

Recall the rejection method:

$$|\vec{a}\cdot\vec{\lambda}| \sim U_{[0,1]}$$
$$when \quad \vec{\lambda} \sim U_{S_2}$$

1. Alice picks $\vec{\lambda}_0 \sim U_{S_2}$

2. Alice picks $u_0 \sim U_{[0,1]}$

$$u_0 = |\vec{a}\cdot\vec{\lambda}_1| \sim U_{[0,1]}$$

3. Test whether $u_0 = |\vec{a}\cdot\vec{\lambda}_1| \leq |\vec{a}\cdot\vec{\lambda}_0|$

If **Test OK** Alice **ACCEPTS** $\vec{\lambda}_0$ and sets $\vec{\lambda}_s = \vec{\lambda}_0$

Otherwise Alice **REJECTS** $\vec{\lambda}_0$ go back to 1 with another

So, Alice has $\vec{\lambda}_s \sim |\vec{a}\cdot\vec{\lambda}_s| / 2\pi$

Local Sampling of the biased distribution: a new method

The Choice method:

$$|\vec{a}\cdot\vec{\lambda}| \sim U_{[0,1]}$$
$$when \quad \vec{\lambda} \sim U_{S_2}$$

1. **Alice picks** $\vec{\lambda}_0 \sim U_{S_2}$

2. **Alice picks** $u_0 \sim U_{[0,1]}$

$$u_0 = |\vec{a}\cdot\vec{\lambda}_1| \sim U_{[0,1]}$$

3. **Test whether** $u_0 = |\vec{a}\cdot\vec{\lambda}_1| \leq |\vec{a}\cdot\vec{\lambda}_0|$

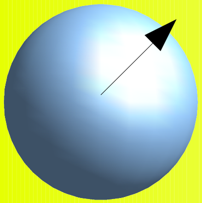
If **Test OK** Alice **ACCEPTS** $\vec{\lambda}_0$ and sets $\vec{\lambda}_s = \vec{\lambda}_0$

Otherwise Alice **ACCEPTS** $\vec{\lambda}_1$ and sets $\vec{\lambda}_s = \vec{\lambda}_1$

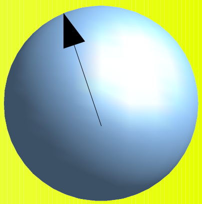
So, Alice has $\vec{\lambda}_s \sim |\vec{a}\cdot\vec{\lambda}_s|/2\pi$

Step 2: Distributed Sampling problem With communication

$$\vec{\lambda}_0, \vec{\lambda}_1 \sim U_{S_2}$$



$\vec{a}$



$\vec{b}$

Alice

Bob

Choice method :

1. Alice picks $\vec{\lambda}_0 \sim U_{S_2}$

2. Alice picks $\vec{\lambda}_1 \sim U_{S_2}$

3. Tests whether

$$\left|\vec{a}\cdot\vec{\lambda}_1\right| \le \left|\vec{a}\cdot\vec{\lambda}_0\right|$$

If yes $\vec{\lambda}_s = \vec{\lambda}_0$

If no $\vec{\lambda}_s = \vec{\lambda}_1$

x = 0 or 1

Communication

1 bit Worst Case [TB03]

Bob sets :

$$\vec{\lambda}_s = \vec{\lambda}_x$$

$\vec{\lambda}_s$

$$With \quad \vec{\lambda}_s \sim \frac{\left|\vec{a}\cdot\vec{\lambda}_s\right|}{2\pi}$$

$\vec{\lambda}_s$

Without resource

$$\vec{\lambda}_0, \vec{\lambda}_1 \sim U_{S_2}$$

$\vec{a}$ $\vec{b}$

Choice method :

Alice picks $\vec{\lambda}_0, \vec{\lambda}_1$

Tests whether

$$|\vec{a}\cdot\vec{\lambda}_1| \leq |\vec{a}\cdot\vec{\lambda}_0|$$

If yes $\vec{\lambda}_s = \vec{\lambda}_0$

x=0

If no $\vec{\lambda}_s = \vec{\lambda}_1$

x=1

Alice

Bob

Bob always sets: $\vec{\lambda}_s = \vec{\lambda}_0$

Simulation of non separable Werner State.

$$W = p\ |\psi^- \times \psi^-| + (1-p)\ \frac{1}{4}$$ ***With p=1/2***

Alice's output :

$$A(\vec{a}, \vec{\lambda}_s) = \quad -sign(\vec{a}\cdot\vec{\lambda}_s)$$

Bob's output :

$$B(\vec{b}, \vec{\lambda}_s) = \quad sign(\vec{b}\cdot\vec{\lambda}_s)$$

With a non local box

$$\vec{\lambda}_0 , \vec{\lambda}_1 \sim U_{S_2}$$

$\vec{a}$

$\vec{b}$

Choice method :

Alice picks $\vec{\lambda}_0 , \vec{\lambda}_1$

Tests whether

$$|\vec{a}\cdot\vec{\lambda}_1| \leq |\vec{a}\cdot\vec{\lambda}_0|$$

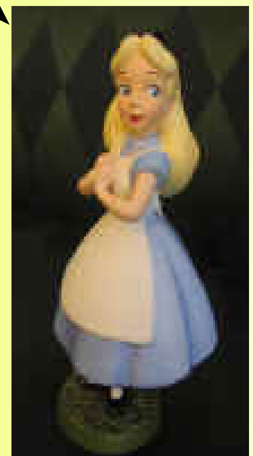
If yes $\vec{\lambda}_s = \vec{\lambda}_0$

x=0

If no $\vec{\lambda}_s = \vec{\lambda}_1$

x=1

Alice



Bob

Bob always sets: $\vec{\lambda}_s = \vec{\lambda}_0$

Bob Tests whether

$$sign(\vec{b}\cdot\vec{\lambda}_0) = sign(\vec{b}\cdot\vec{\lambda}_1)$$

If yes: cool ! *y=0*

If no: Aïe ! *y=1*

x

y

Non-Local Box: $x \wedge y = a \oplus b$

a

b

Alice's output :

$$A(\vec{a},\vec{\lambda}_s) = -(-1)^{a} sign(\vec{a}\cdot\vec{\lambda}_s)$$

Bob's output :

$$B(\vec{b},\vec{\lambda}_s) = (-1)^{b} sign(\vec{b}\cdot\vec{\lambda}_s)$$

Conclusion

- **The distributed sampling problem give us a unified framework for the problem of simulating quantum correlations.**

- **Related results:**

  POVMs : Post Selection(Efficiency 1/3),
  Communication and Non-local Boxes (2 nl-Bits + 4 bits on average)

  Some preliminary results for higher dimensions.

- **Open problems:**

  Multipartite states and non maximally entangled states.